UNIVERSITY OF OXFORD

Transport of driven colloids in optical landscapes

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Outline

Transport of driven colloids in optical landscapes

- **Synchronisation: dynamic mode locking**
  - *1* particle *DC* driven
  - *1* particle *DC* + *AC* driven: dynamic mode locking
  - *N* particles *DC* + *AC* driven: dynamic mode locking of a kink
- **Depinning of finite colloidal chains: Aubry-type transition**
  - *N* particles *DC* driven

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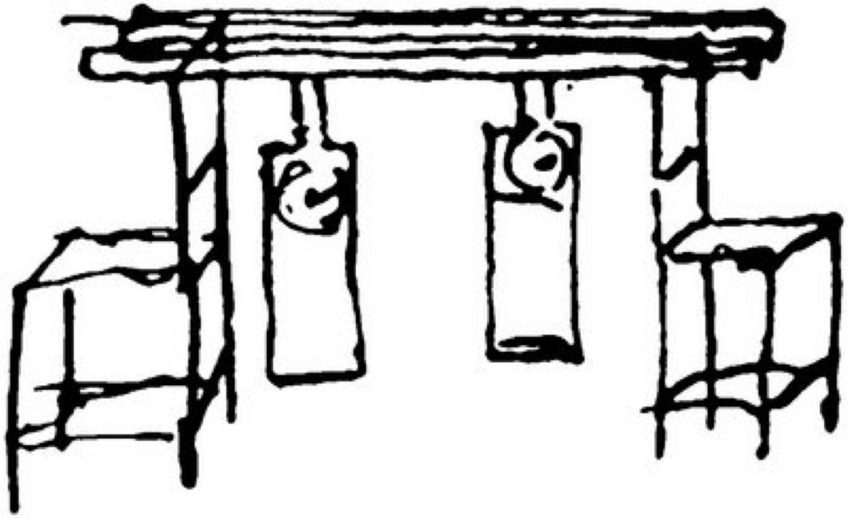
Synchronisation

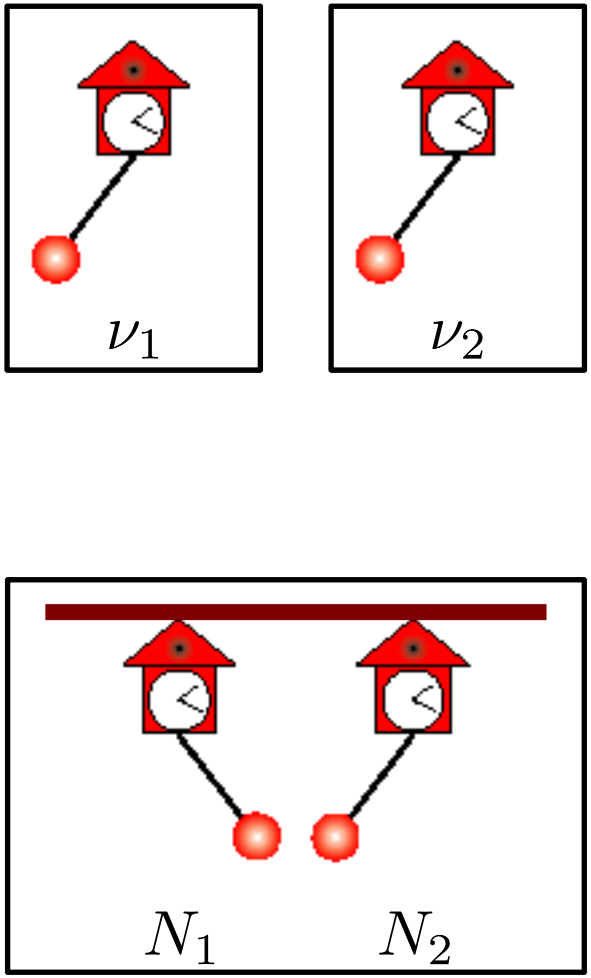
The adjustment of rhythms due to an interaction is the essence of synchronisation

Christiaan Huygens (1629-1695)



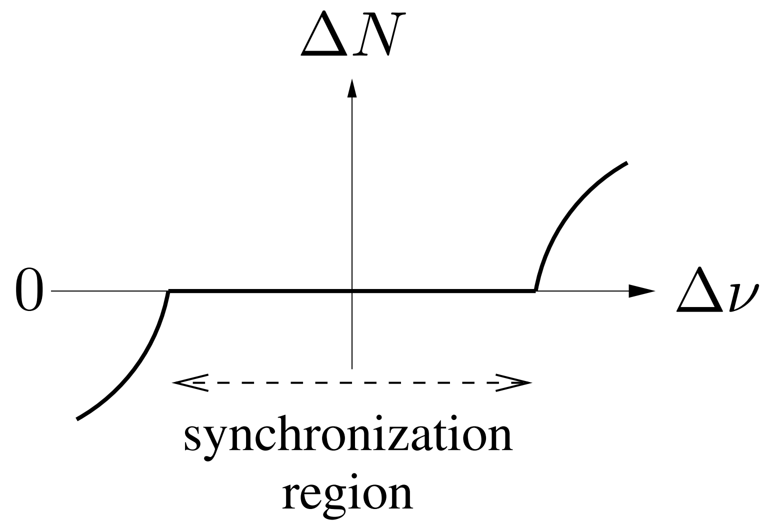
... we suspended two clocks so constructed from two hooks imbedded in the same wooden beam ... the motions of each pendulum in opposite swings were so much in agreement that they never receded the least bit from each other ... the cause of this is due to the motion of the beam, even though this is hardly perceptible.

Synchronisation of coupled oscillators



$$\Delta\nu = \nu_1 - \nu_2$$

$$\Delta N = N_1 - N_2$$

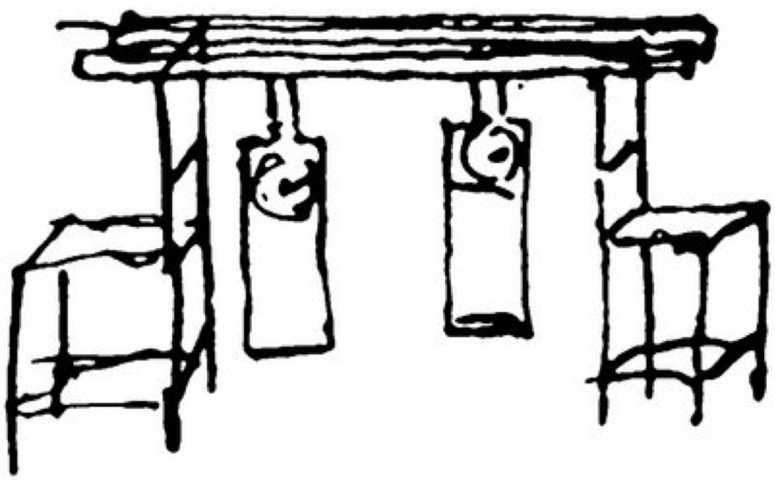


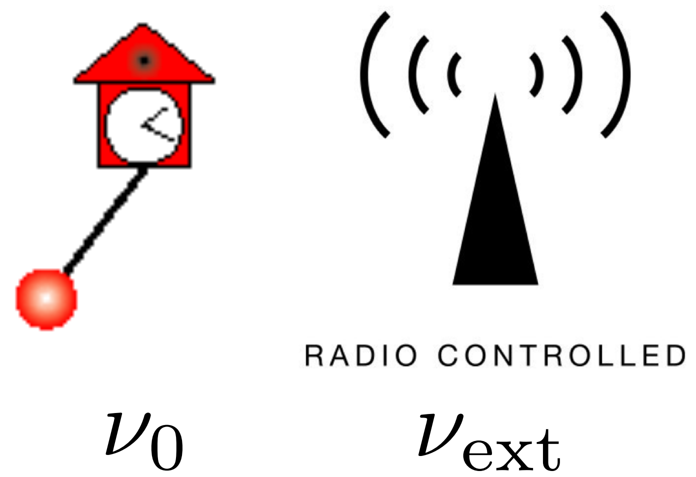
Pikovsky, Rosenblum, and Kurths, *Synchronization*, Cambridge University Press (2001)

Synchronisation by external forcing

Huygens in the 21st century

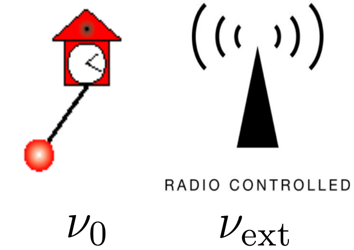
Radio-controlled clocks: relatively non-precise clocks are made perfect by being adjusted by a periodic radio signal

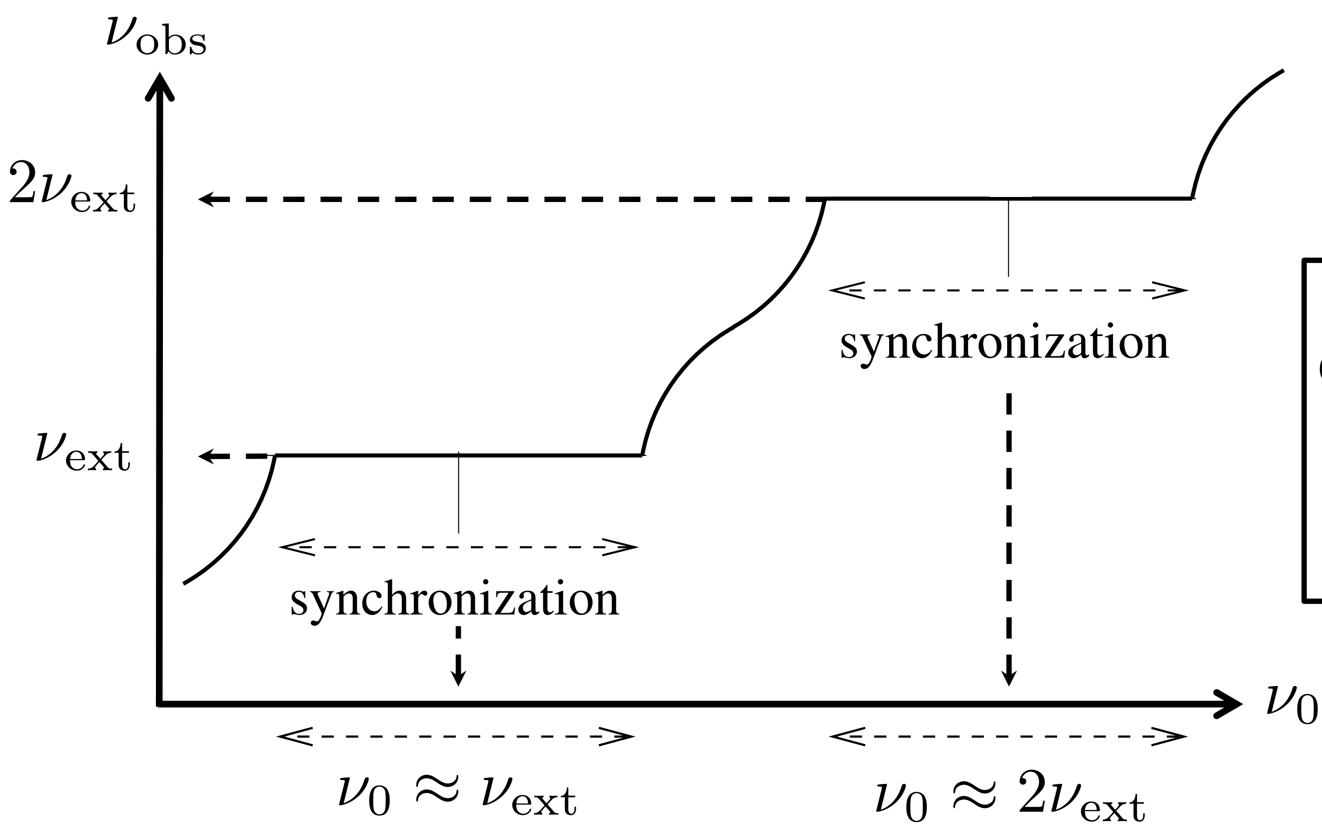




Synchronisation by external forcing

- Oscillator (ν_0) synchronises to external modulation (ν_{ext})





Synchronisation = dynamic mode locking

$$\nu_0 \approx n\nu_{ext}$$

$$\nu_{obs} = n\nu_{ext}$$

Dynamic mode locking

- Driven Josephson junctions (Shapiro steps)
- Charge density waves
- Vortex lattices
- Ring laser gyros
- ...

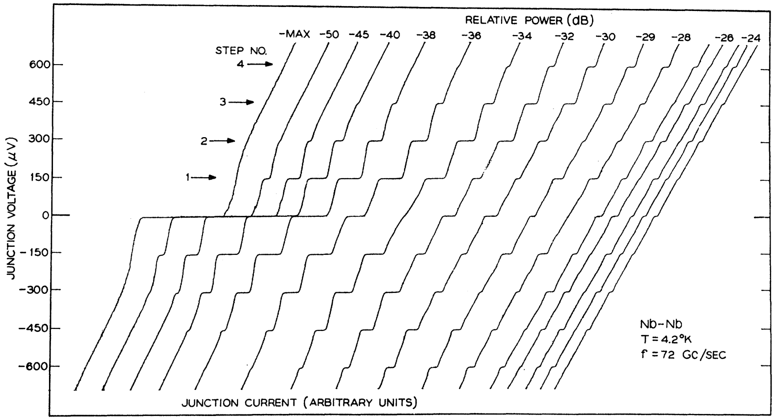


FIG. 1. Voltage-current curves for a Nb-Nb point-contact Josephson junction exposed to a 72-Gc/sec signal at various power levels.

Equation of motion of a driven oscillator

$$\zeta \frac{d\phi}{dt} = F_{ext} + b \sin \phi$$

Only averaged properties studied

Microscopic dynamics difficult to visualise

Dynamic mode locking in driven colloids

Equation of motion of a driven oscillator

$$\zeta \frac{d\phi}{dt} = F_{ext} + b \sin \phi$$

$$\phi \rightarrow x$$

$$\zeta \frac{dx}{dt} = F_{ext} + b \sin x$$

Equation of motion of a driven colloid in a periodic potential

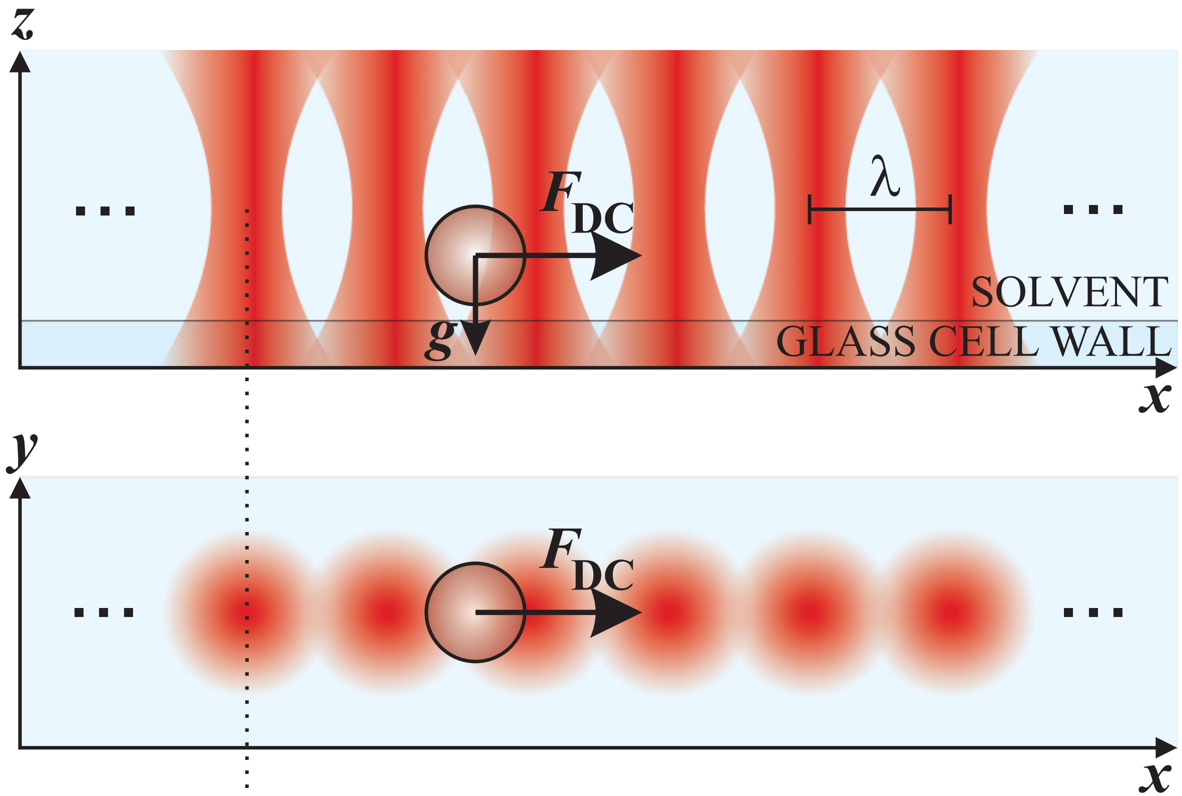
Microscopic dynamics

Tunable interactions

THIS WORK

Experimental details

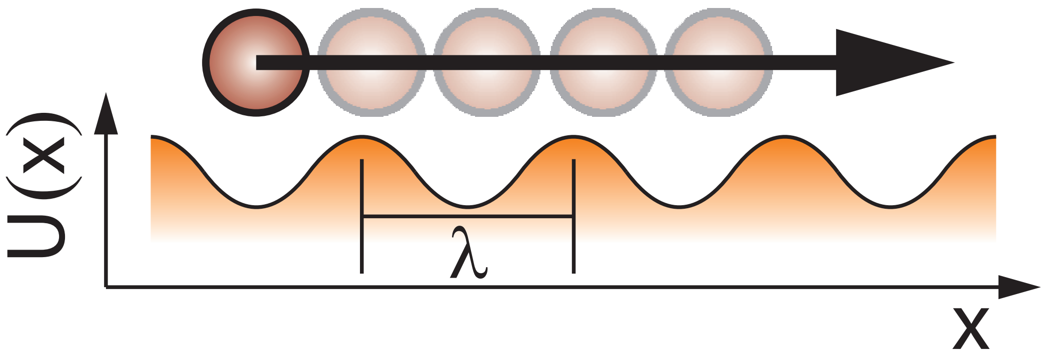
- 3 µm diameter polystyrene, paramagnetic particles in 20% $EtOH_{(aq)}$
- 1D sinusoidal potential energy landscape: optical tweezers



Experimental details

- 3 µm diameter polystyrene, paramagnetic particles in 20% $EtOH_{(aq)}$
- 1D sinusoidal potential energy landscape: optical tweezers
- Piezo stage: DC and/or AC driving velocity with frequency ν
- Video-microscopy: obtain particle trajectory $x(t)$
- Trajectory gives average velocity $\overline{v}$ and instantaneous velocity $v(t)$

$$v_{\mathrm{DC}} + v_{\mathrm{AC}} \sin(2\pi\nu t)$$



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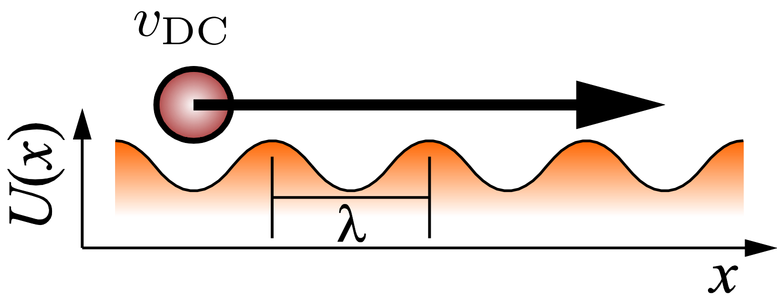
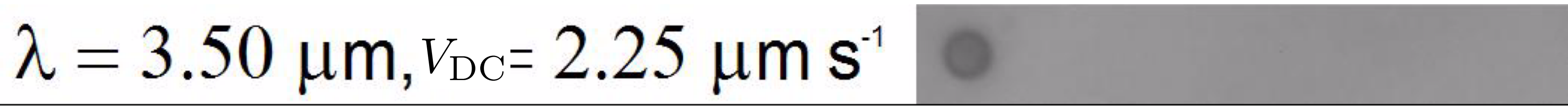
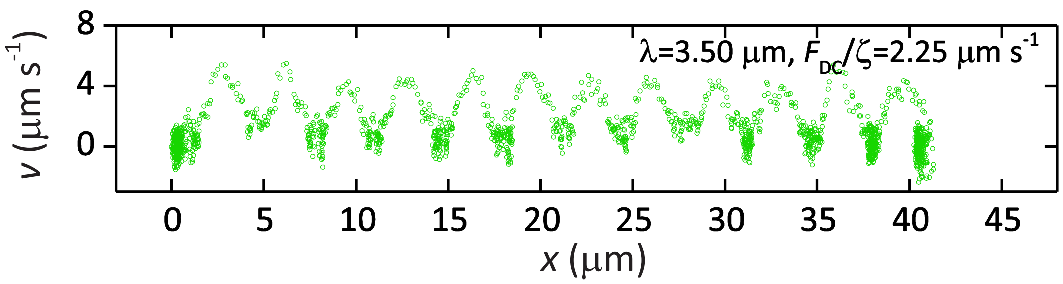
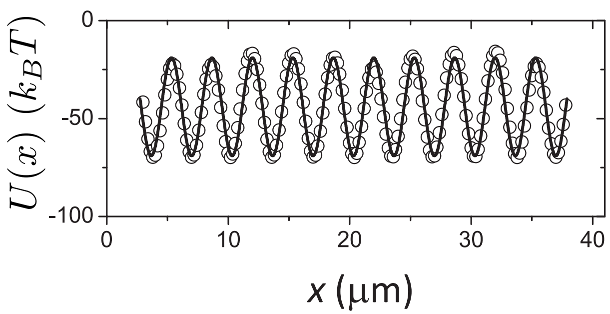
DC drive over a sinusoidal potential



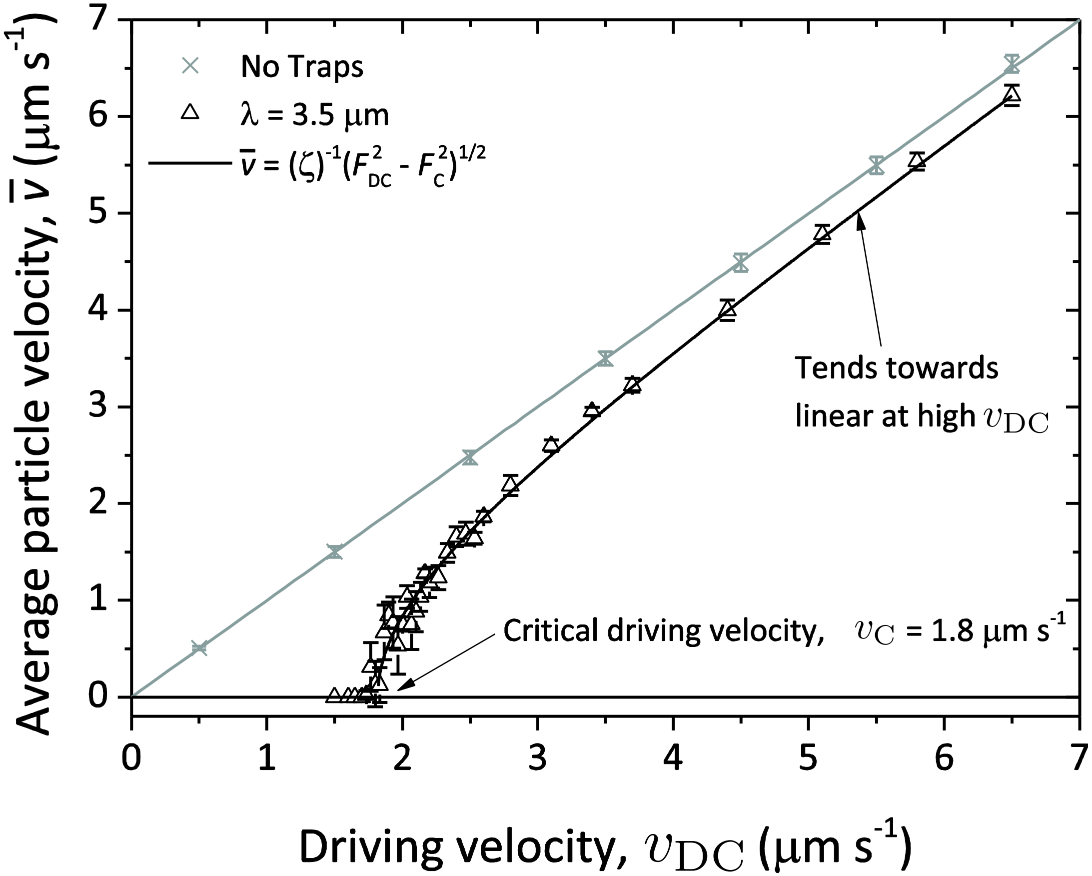






- Sinusoidal optical potential energy landscape: $F_T(x) \sim \sin\left(\frac{2\pi x}{\lambda}\right)$

Average velocity of DC driven particle



$$\frac{dx}{dt} = v_{DC} + \frac{F_T(x)}{\zeta}$$

$$F_T(x) \sim \sin\left(\frac{2\pi x}{\lambda}\right)$$

$$\bar{v} = \sqrt{v_{\mathrm{DC}}^2 - v_{\mathrm{C}}^2}$$

Importantly

DC driven colloid particle = oscillator with frequency $\nu_0 = v_{\mathrm{DC}}/\lambda$

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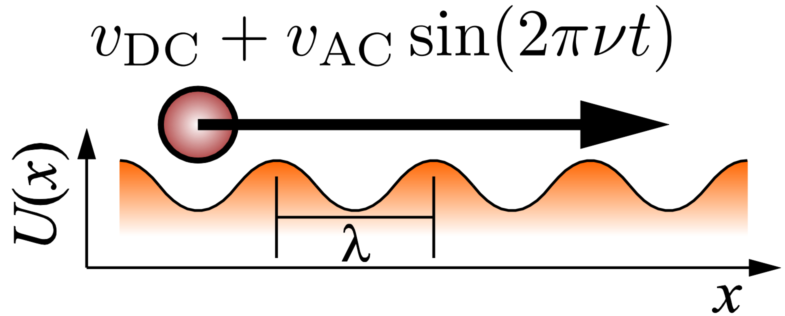
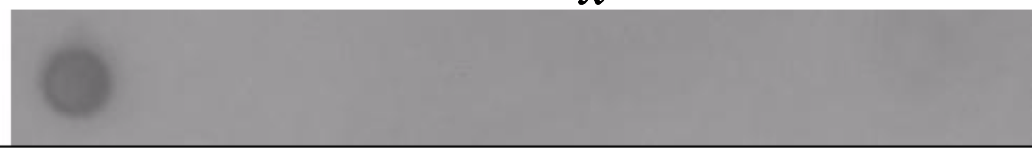
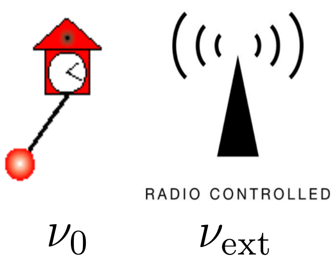
DC + AC drive over a sinusoidal potential



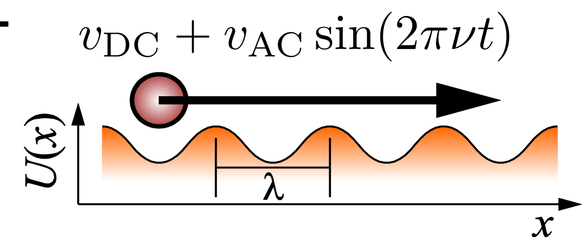
$\lambda = 3.5$ µm $V_{AC} = 5.2$ µm s^{-1}
$\nu = 0.75$ Hz $V_{DC} = 5.833$ µm s^{-1}



dynamic mode locking?

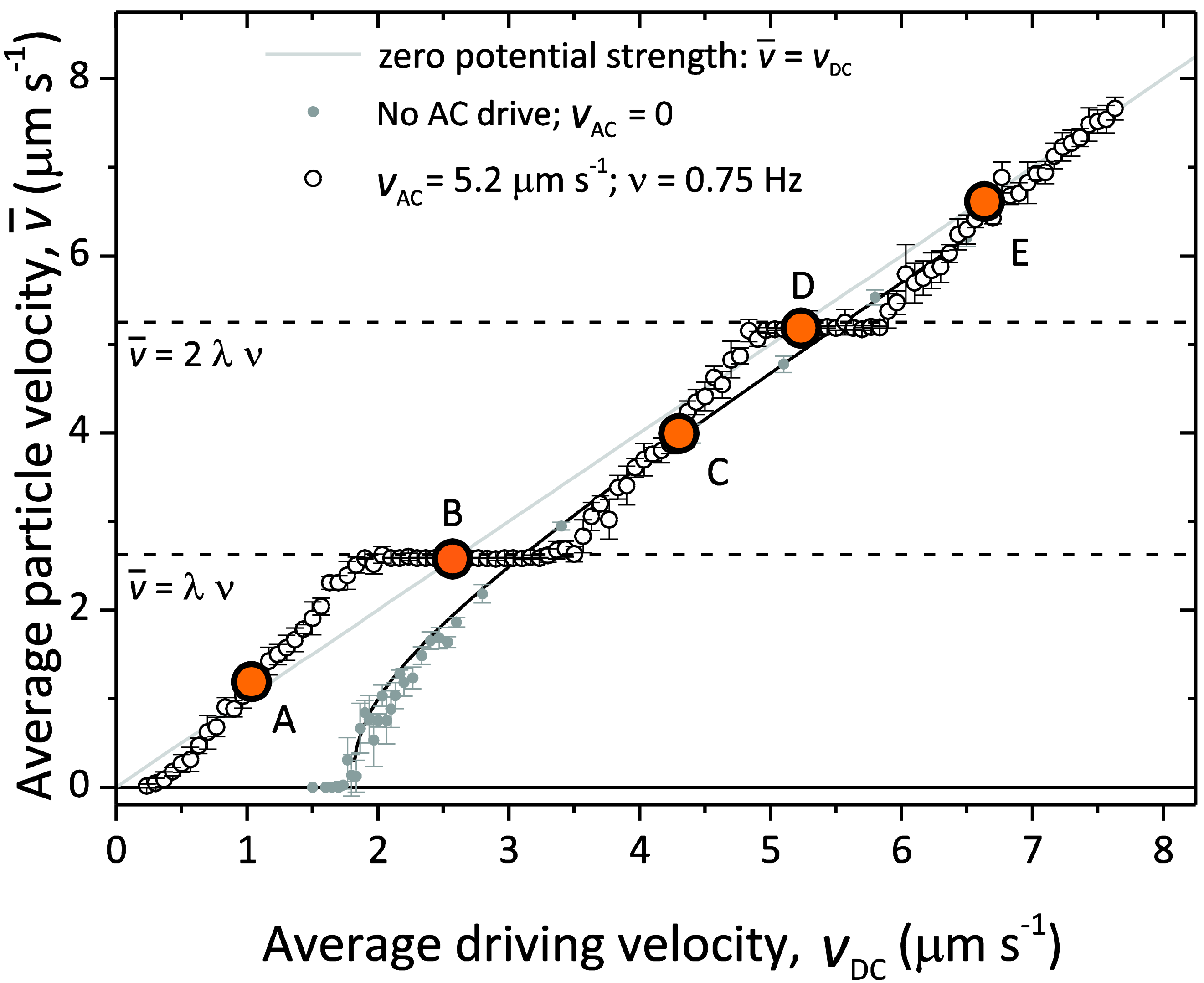


$$\nu_0 \approx n\nu_{ext}$$
$$\nu_{obs} = n\nu_{ext}$$



$$\nu_0 = v_{DC}/\lambda$$
$$\nu_{ext} = \nu n\lambda\nu$$

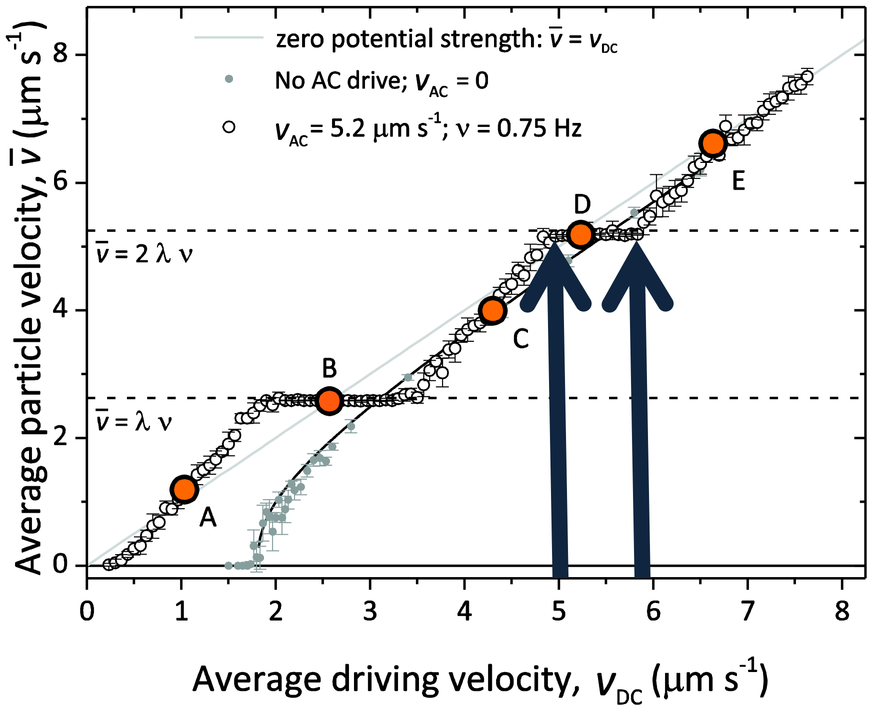
Dynamic mode locking (DC + AC drive)



Synchronisation:

$$\overline{v} = n\lambda\nu$$

“Devil’s staircase”: mode locking steps



$\lambda = 3.5\ \mu m$ $V_{AC} = 5.2\ \mu m\ s^{-1}$
$\nu = 0.75$ Hz $V_{DC} = 5.033\ \mu m\ s^{-1}$

$\lambda = 3.5\ \mu m$ $V_{AC} = 5.2\ \mu m\ s^{-1}$
$\nu = 0.75$ Hz $V_{DC} = 5.833\ \mu m\ s^{-1}$

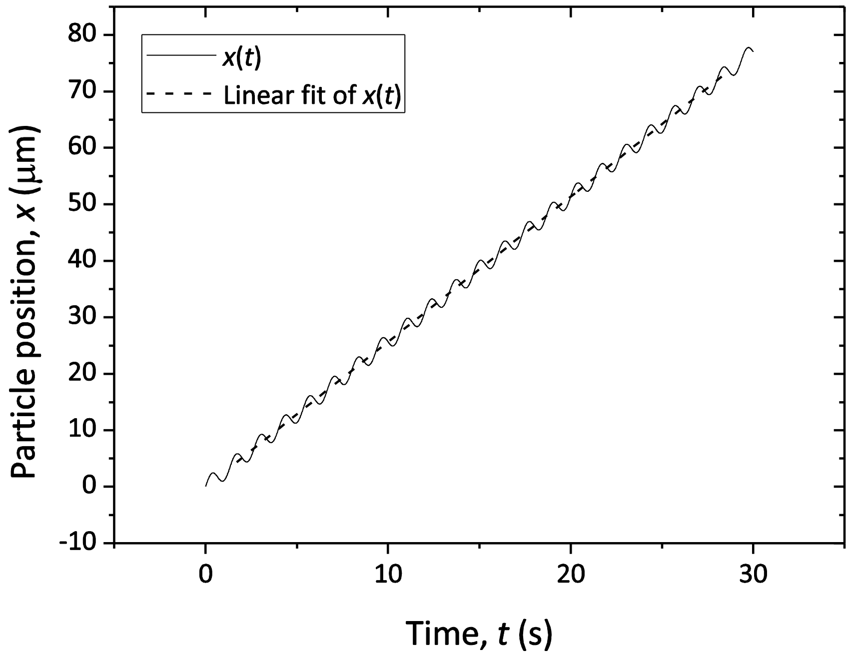
Periodic motion: phase portraits

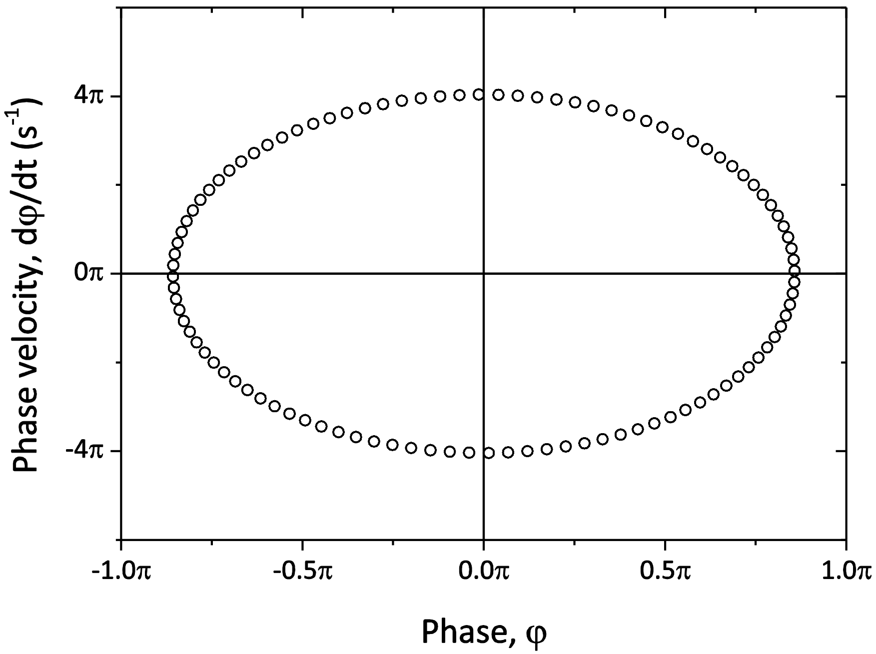
Phase:

$$\varphi = \frac{2\pi}{\lambda}\left(x(t) - \overline{v}t\right)$$

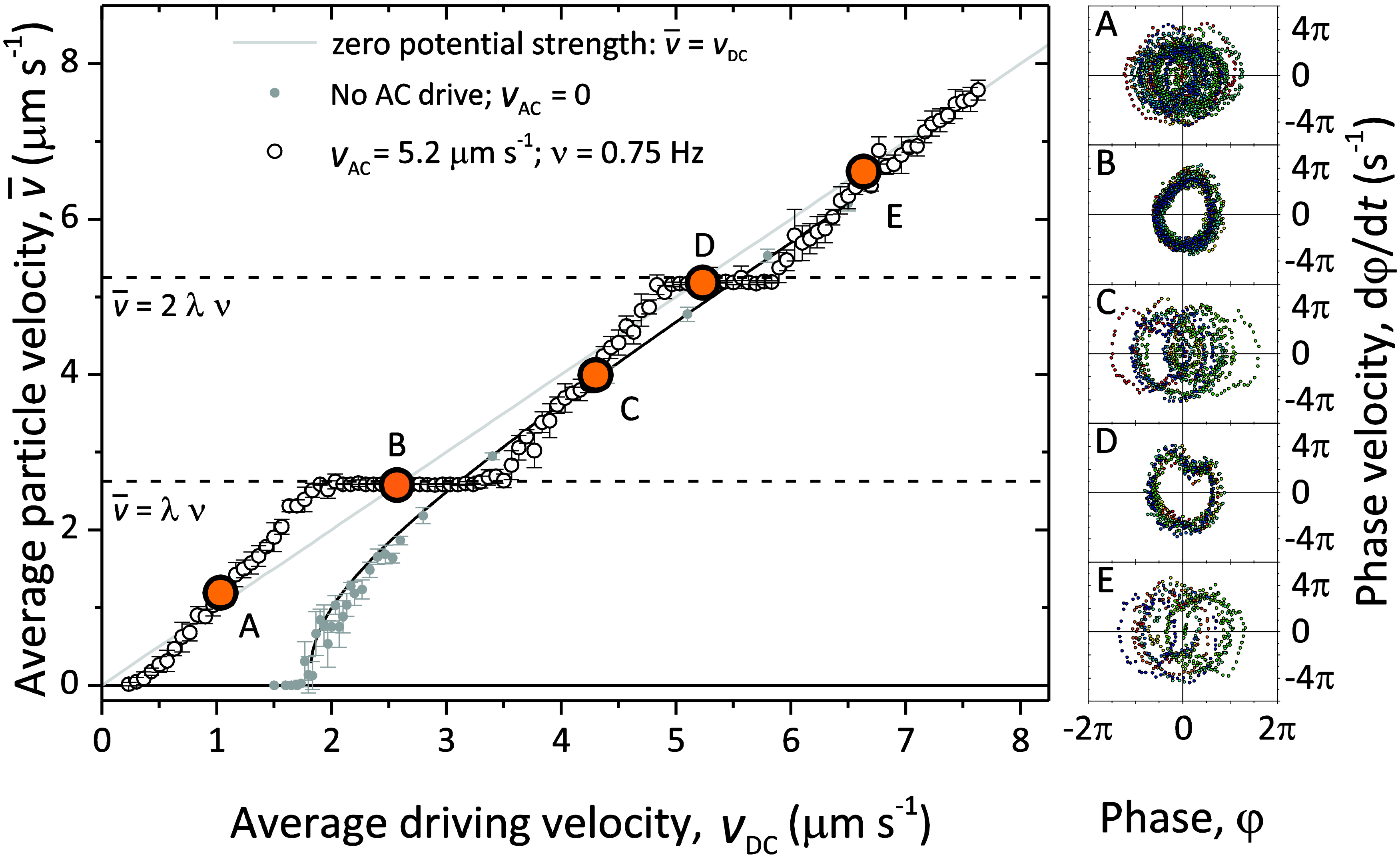
Phase velocity:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{2\pi}{\lambda}\left(v - \overline{v}\right)$$

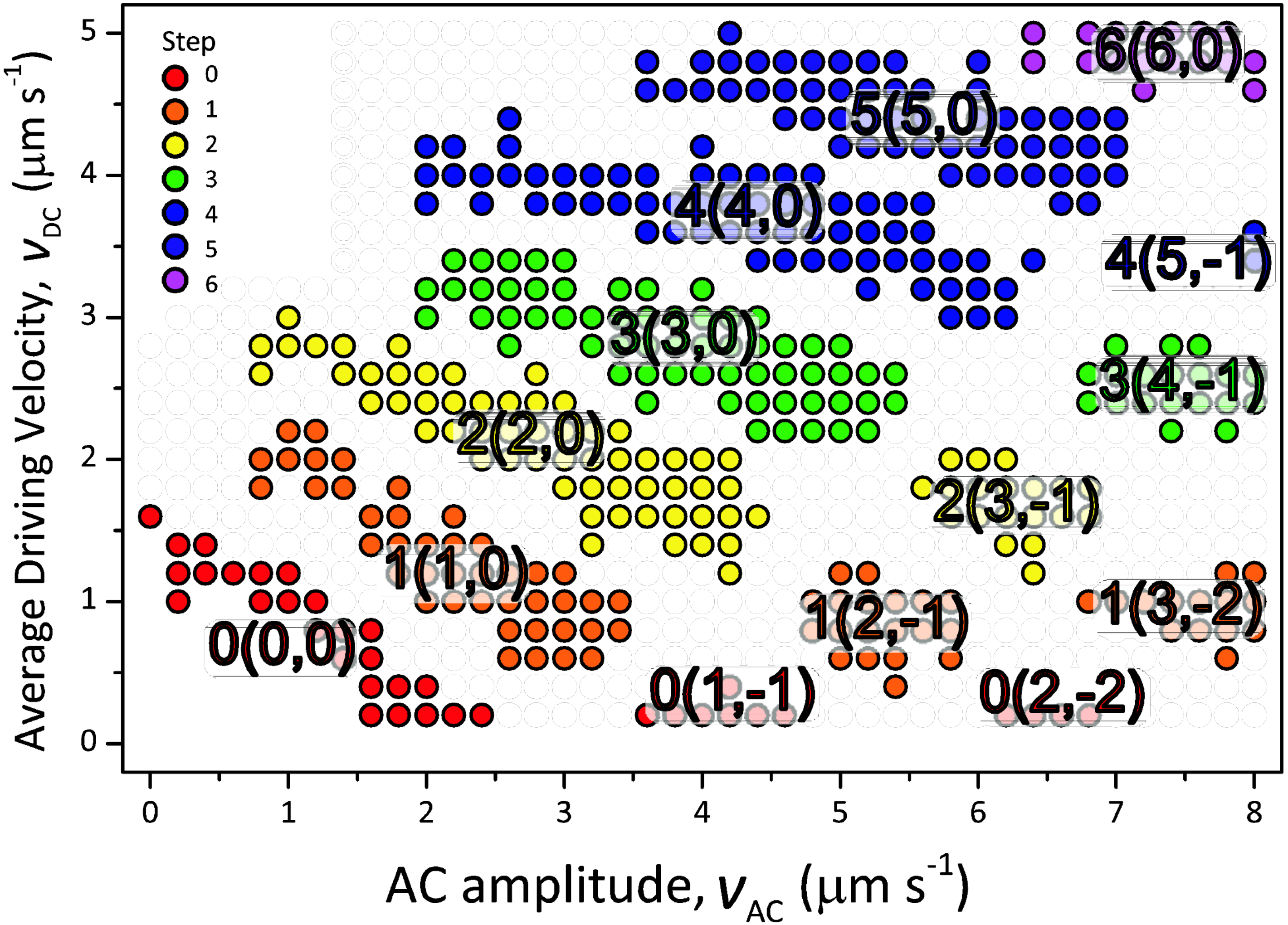




Locked and unlocked states

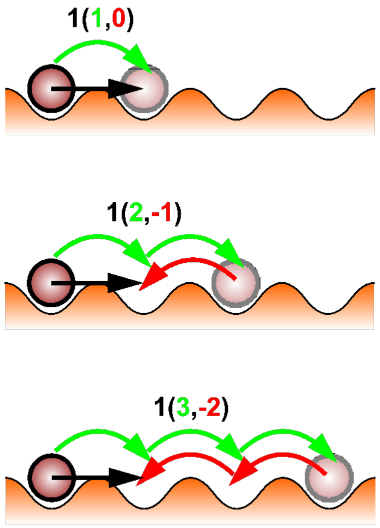


State diagram of locked states

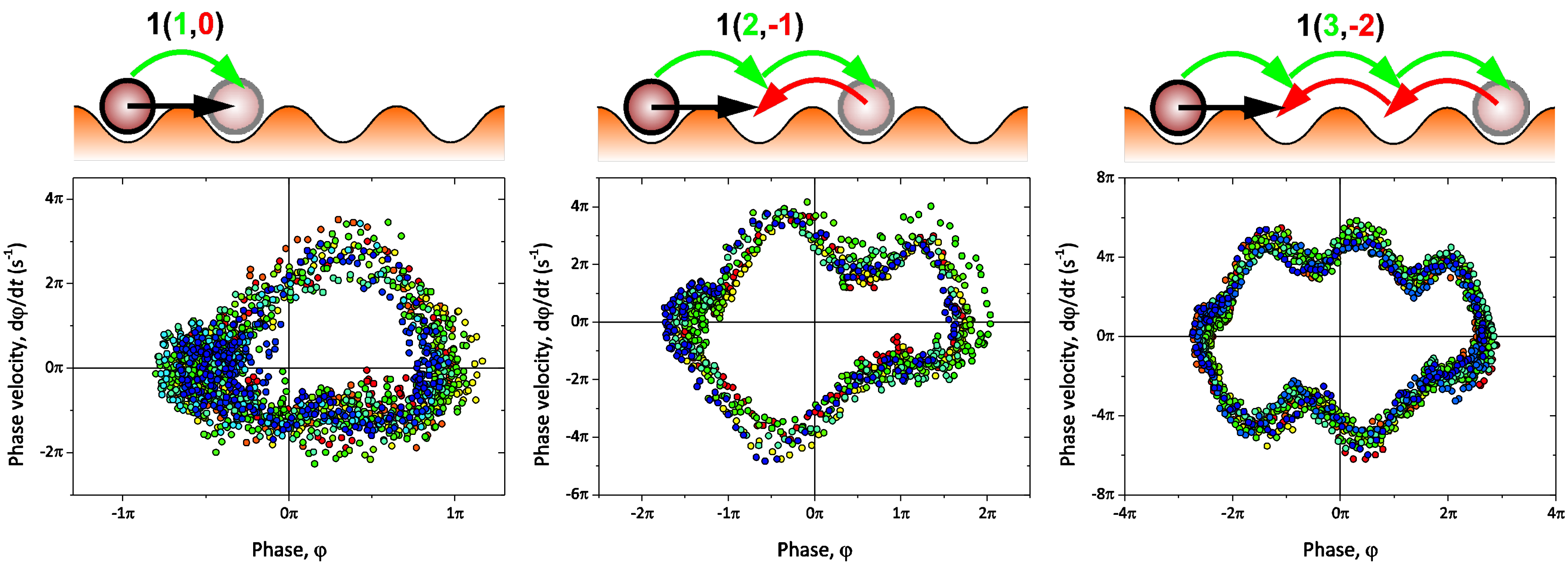


$n(i, j)$:

- n : net λ's moved
- i : positive steps
- j : negative steps



Same average velocity, different modes



- Single colloidal particles show dynamic mode locking behaviour
- Colloidal model system allows access to microscopic details

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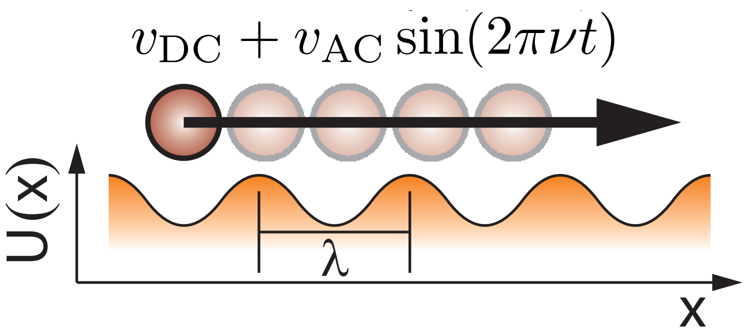
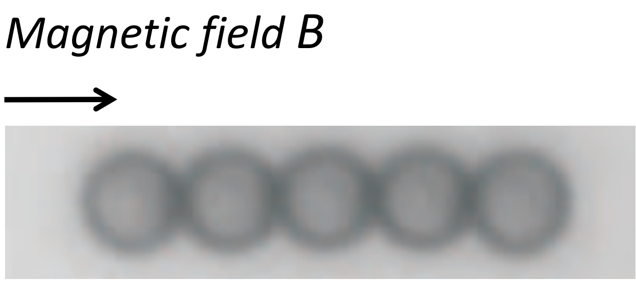
  - *N* particles *DC* driven

Driving a coupled system (*DC* + *AC*)



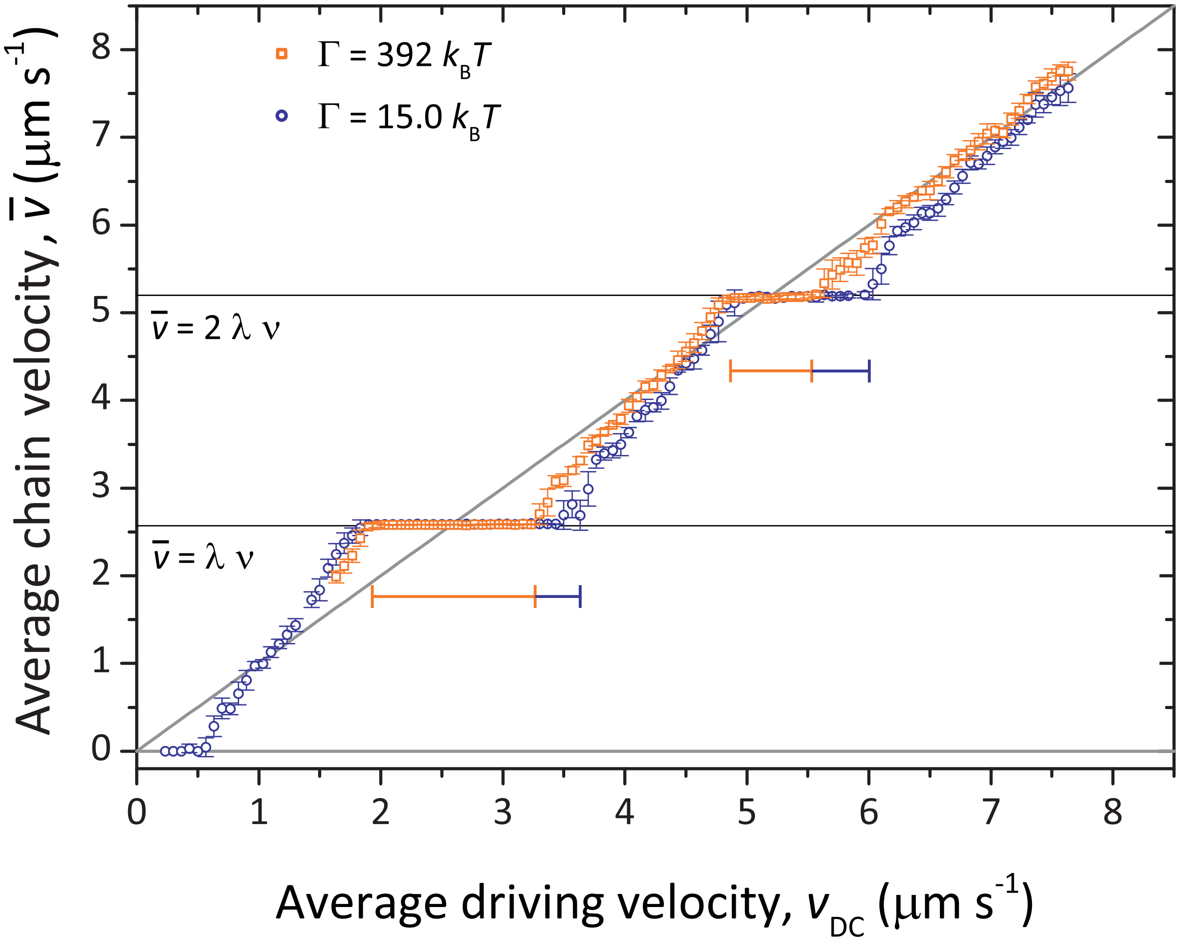


- Chain of 7 magnetically coupled particles (diameter σ = 3 μm)

$$\Gamma = \frac{U(r=\sigma)}{k_B T} \sim \frac{B^2}{\sigma^3}$$

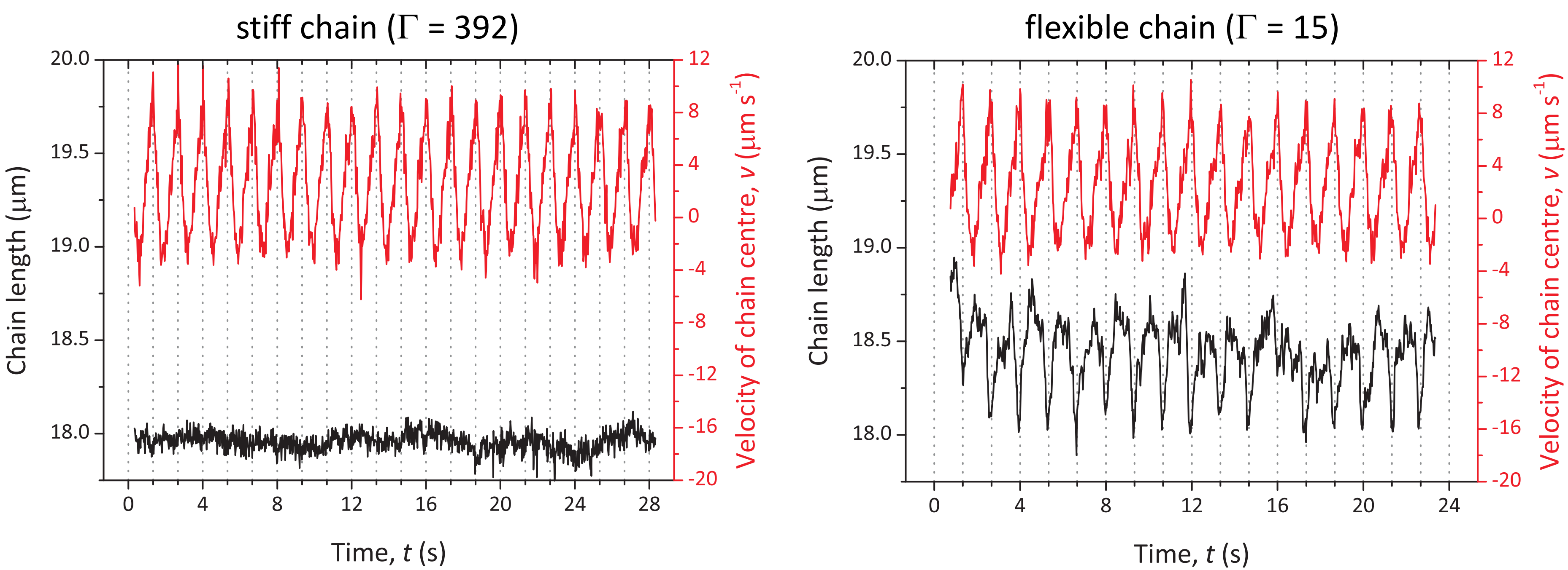
- Flexible chain (Γ = 15) and stiff chain (Γ = 392)
- Sinusoidal landscape λ = 3.5 μm
- Chain position = mean of coordinates of terminal particles

Flexible chain: enhanced mode locking



- 1st (10) step 16% and 2nd (20) step almost 50% wider for flexible chain

Chain length oscillations

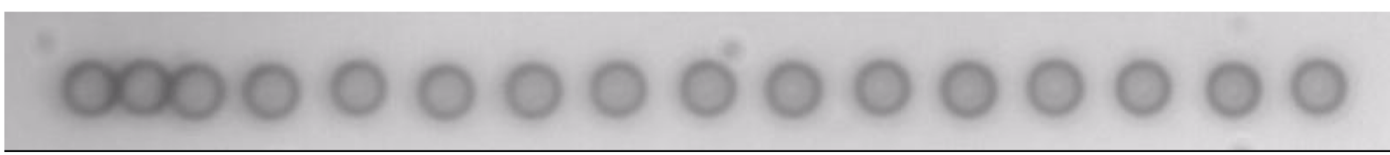


- Flexible chain: chain length and velocity oscillate out of phase
- Breathing mode: density wave (or kink) traveling through mobile chain
- Kink velocity much faster than chain velocity ⟶ hard to resolve

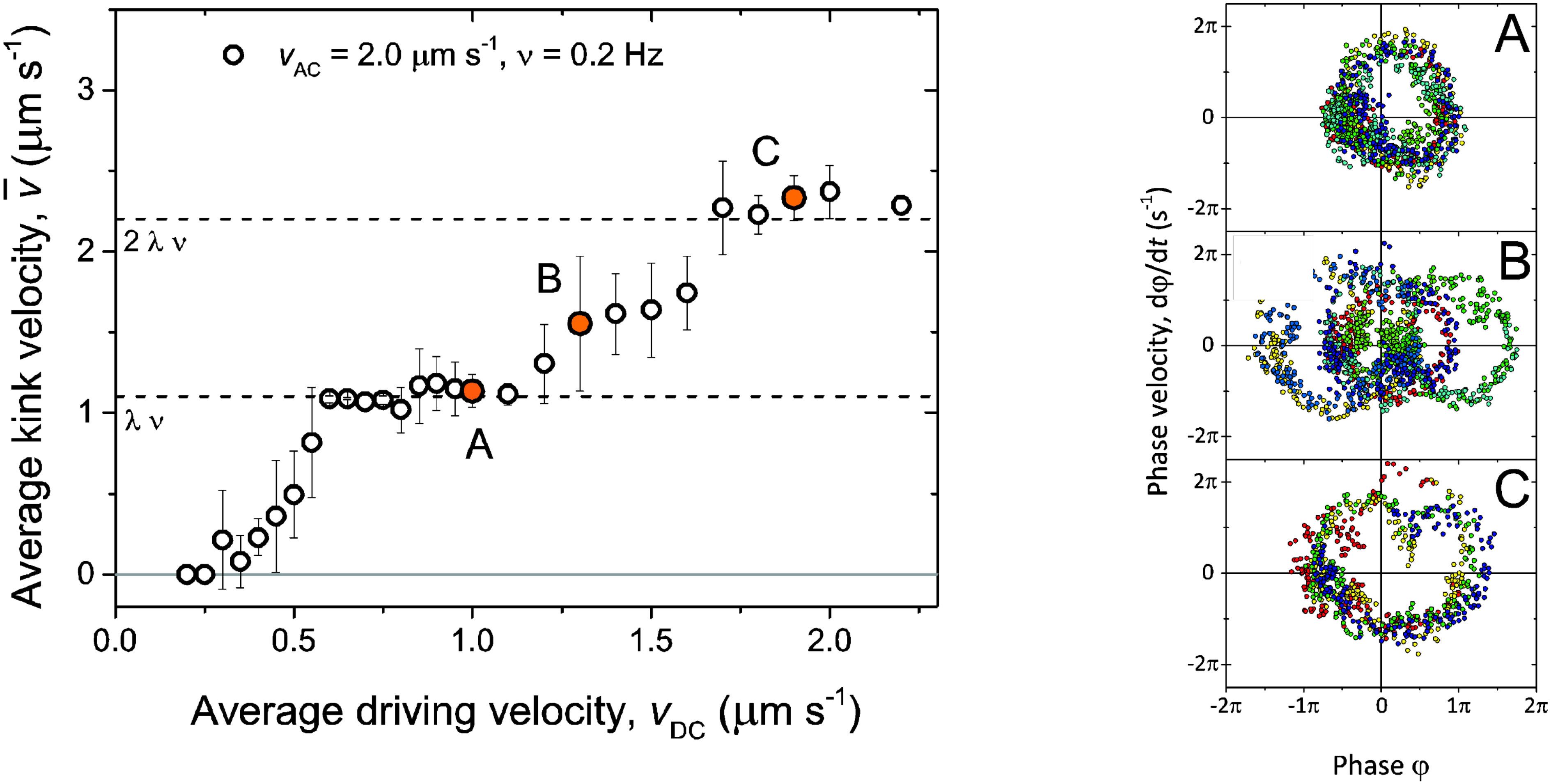
Visualising a *DC* + *AC* driven *artificial* kink in a pinned chain

Driving velocity, $v_{DC} + v_{AC}\sin(2\pi\nu t)$ ⟶ Magnetic field direction ⟶

- 15 strong optical traps, 16 particles
- Extra particle displaces others generating a kink
- Weak magnetic field holds particles in line
- Tracking the kink



Mode locking of a kink



- Confirms dynamic mode locking of a traveling density wave (or kink)

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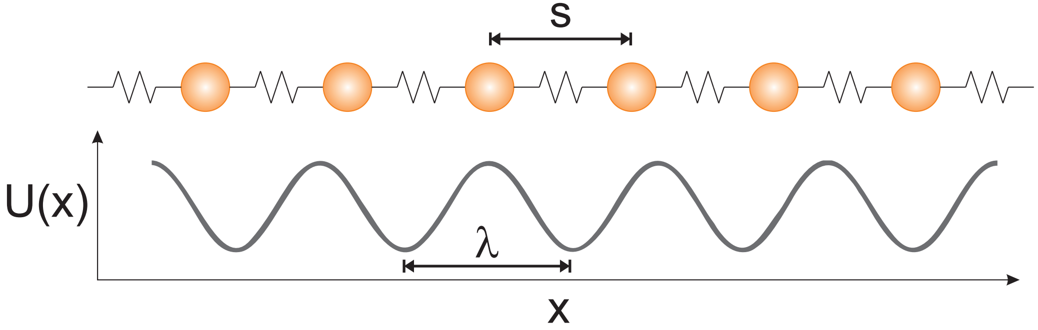
- **Depinning of finite colloidal chains: Aubry-type transition**
  - *N* particles *DC* driven

Aubry transition

Superlubricity due to incommensurate competing length scales

Serge Aubry



At a critical value of the substrate potential, an infinitely long 1D chain incommensurate with the substrate undergoes a transition from a pinned to a free-sliding state.

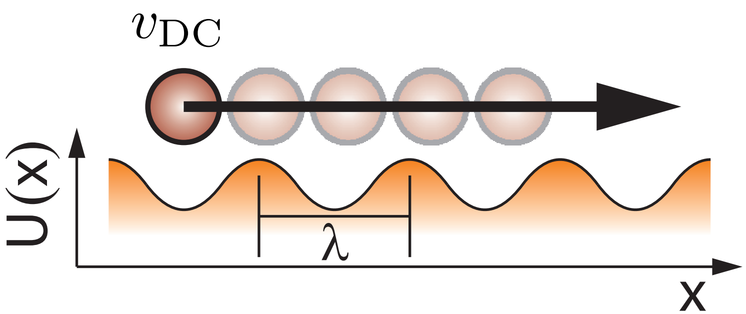
Nanofriction in cold ion traps: A. Benassi et al, Nat. Commun. 2, 236 (2011)

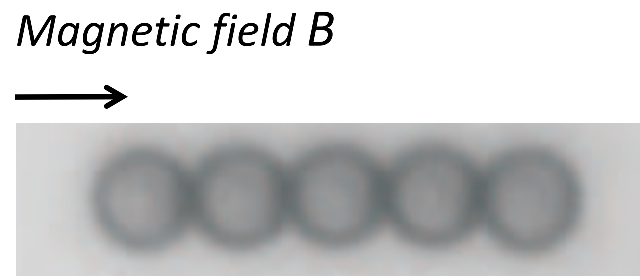
Aubry-type transition in cold atoms: A. Bylinskii et al, Nat. Mater. 15, 717 (2016)

Aubry transition in 2D colloidal monolayers: T. Brazda et al, Phys. Rev. X 8, 011050 (2018)

M. Peyrard and S. Aubry, J. Phys. C: Solid State Phys. 16, 1593 (1983)

Driven finite colloidal chains (*DC*)



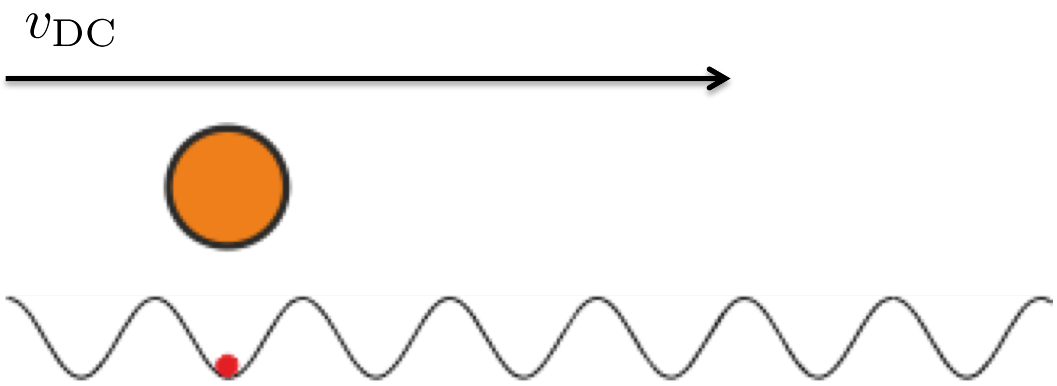


- Sinusoidal landscape λ varied between 3 and 3.9 μm, *fixed depth*
- Chain of *N* = 1 - 25 magnetically coupled particles (diameter σ = 3 μm)

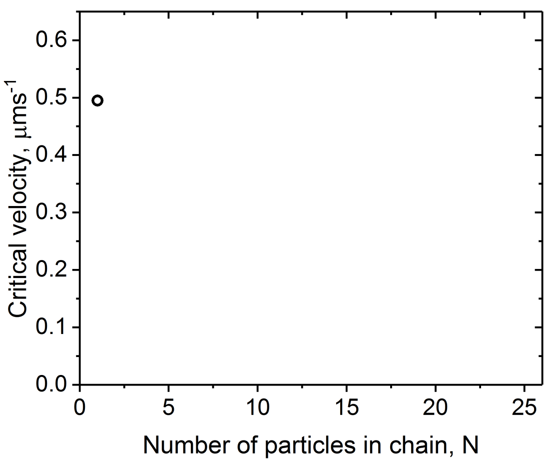
$$\Gamma = \frac{U(r=\sigma)}{k_B T} \sim \frac{B^2}{\sigma^3}$$

- Fixed chain stiffness (Γ = 174 – not too stiff and too flexible)
- Measure critical driving velocity upon increasing *N* with resolution ~fN

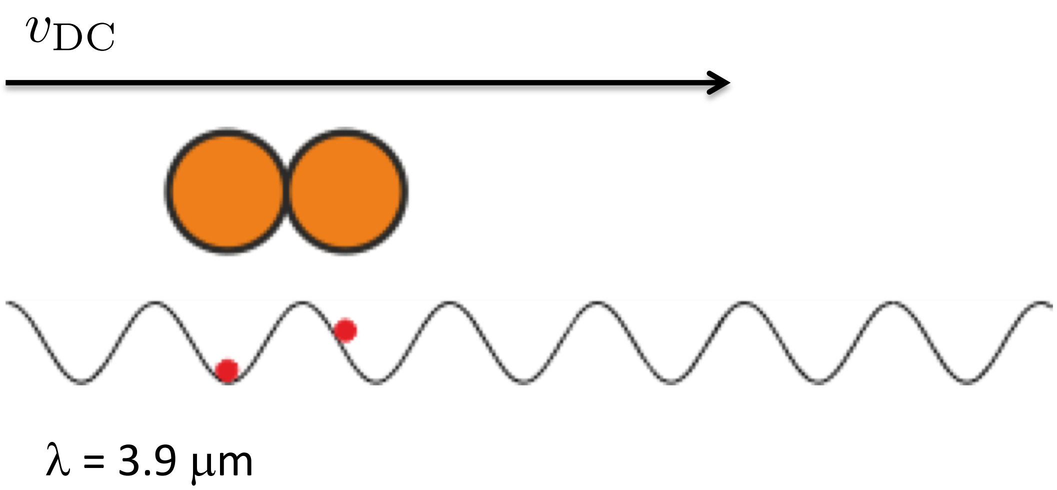
Critical velocity for a single particle

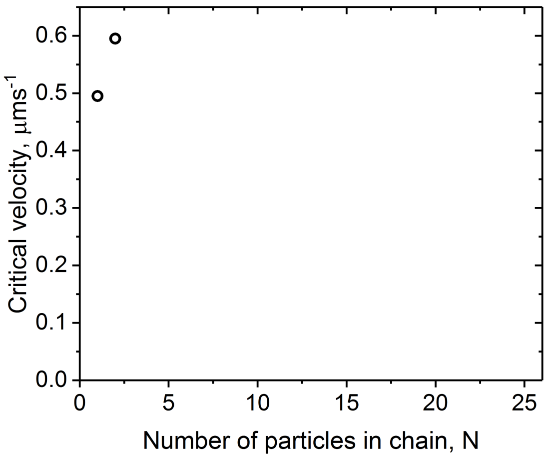


λ = 3.9 μm

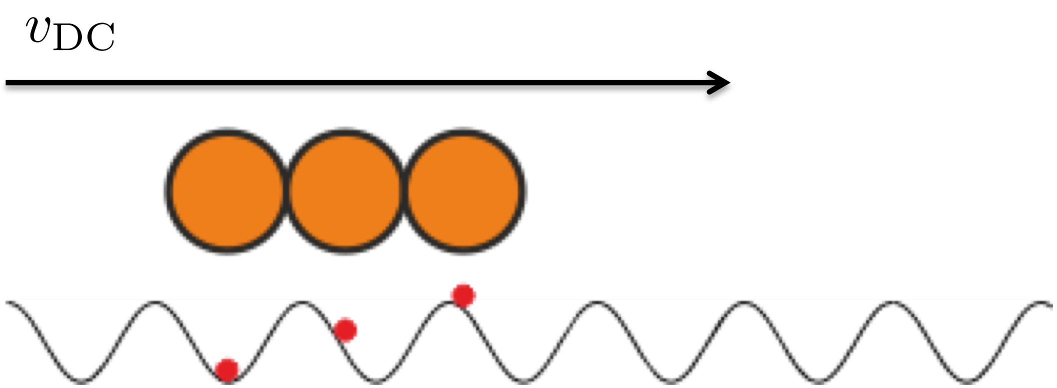


Critical velocity for N = 2

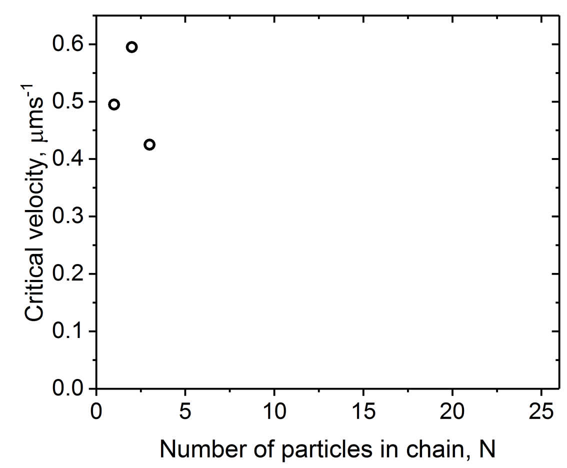




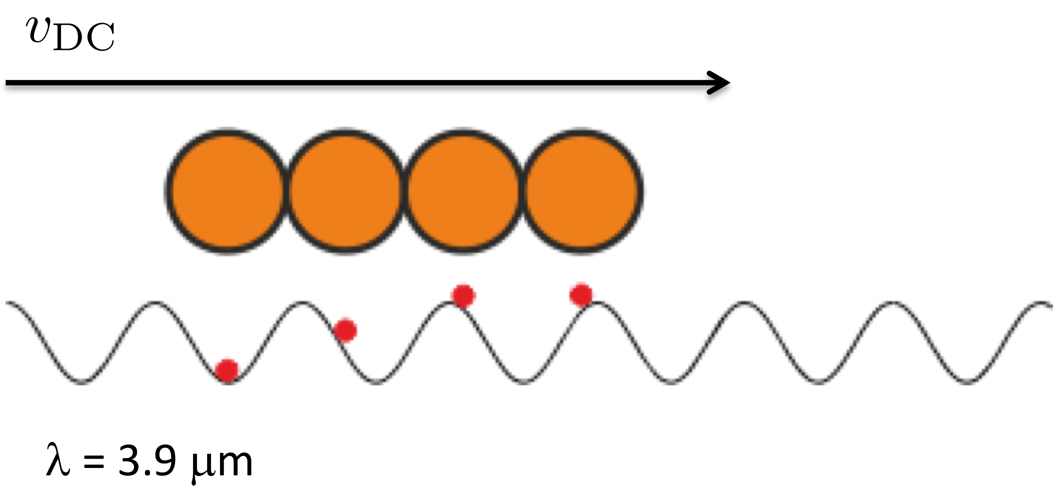
Critical velocity for *N* = 3

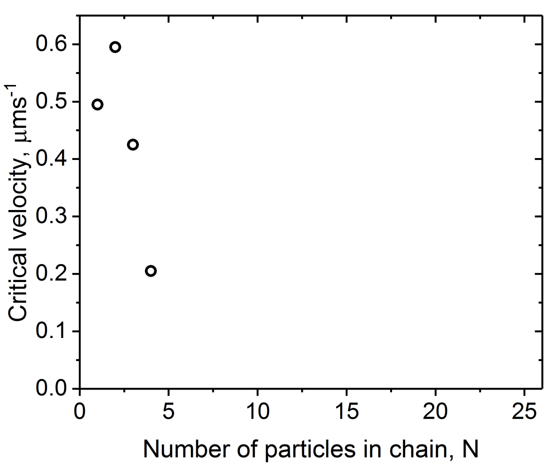


λ = 3.9 μm

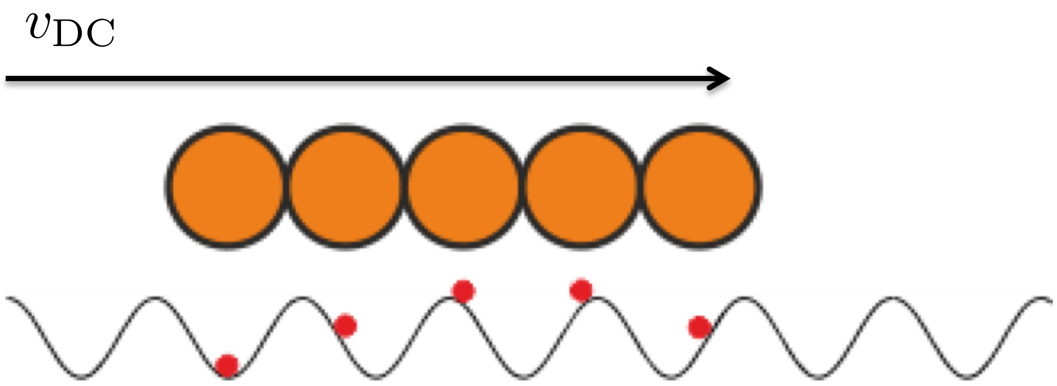


Critical velocity for N = 4

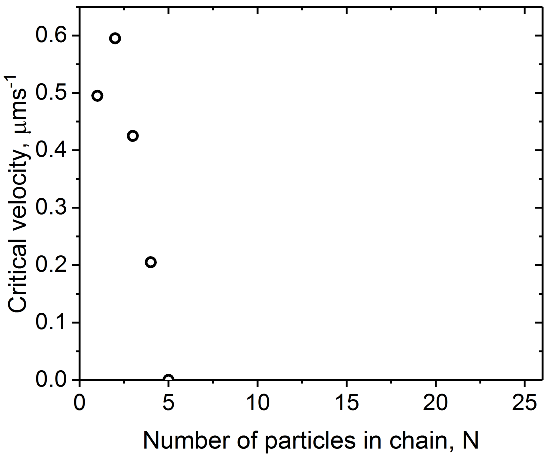




Critical velocity for N = 5

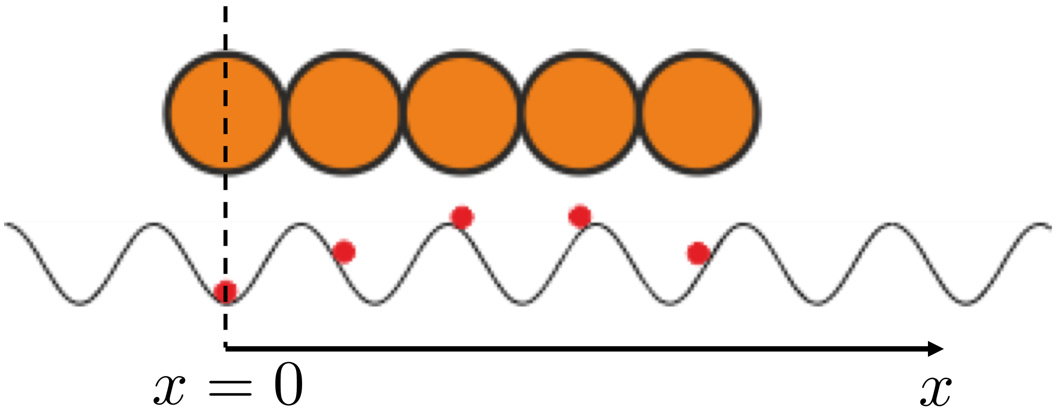


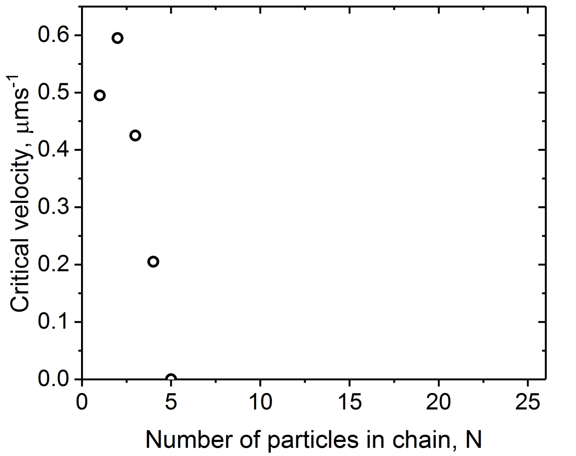
λ = 3.9 μm



- At N =5 critical driving velocity vanishes ...

Critical velocity for *N* = 5

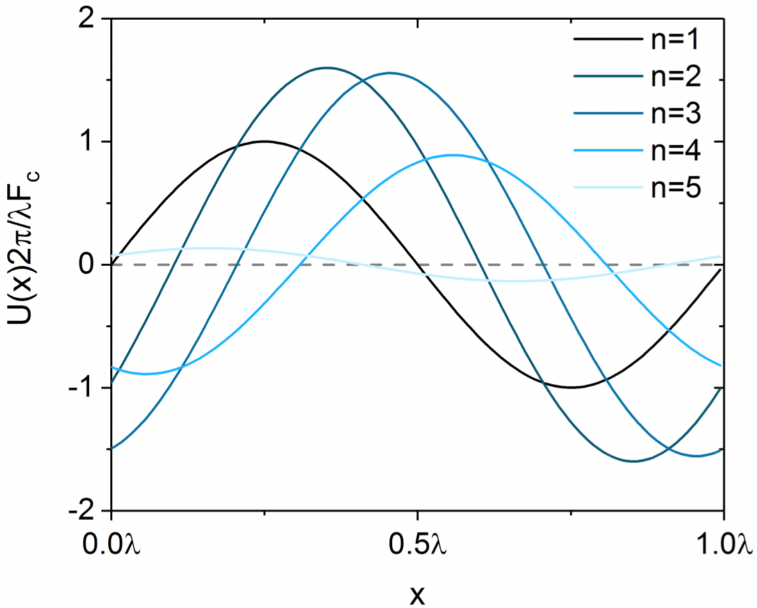




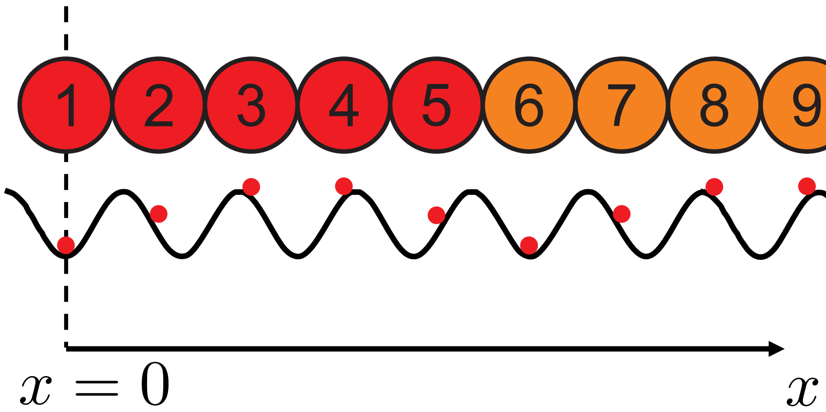
Potential energy of chain as f(x)

$$U_n(x) = -\frac{\lambda F_C}{2\pi}\sum_{j=1}^{n}\cos\left[\left(\frac{2\pi x}{\lambda}\right) - \left(\frac{2\pi s}{\lambda}(j-1)\right)\right]$$

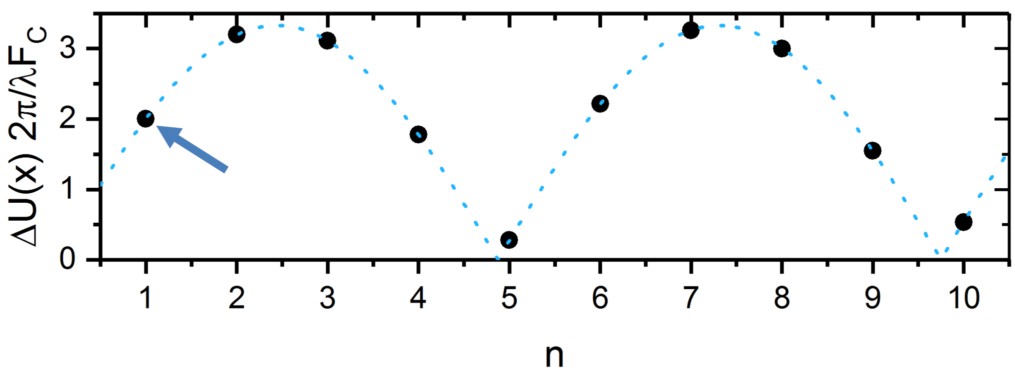
- Assuming fixed interparticle spacing *s*
- *N* = *5*: ***'no' change*** of *U* with respect to *x*

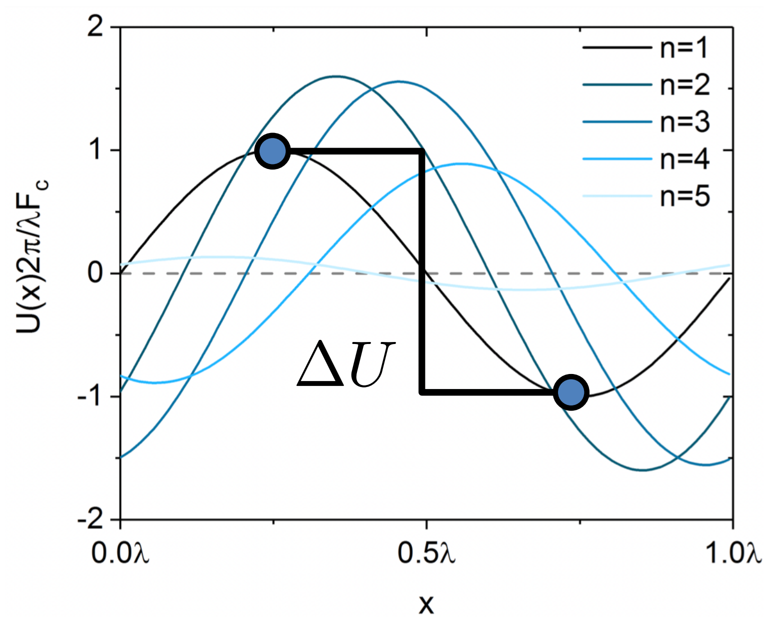


Critical velocity for $N > 5$



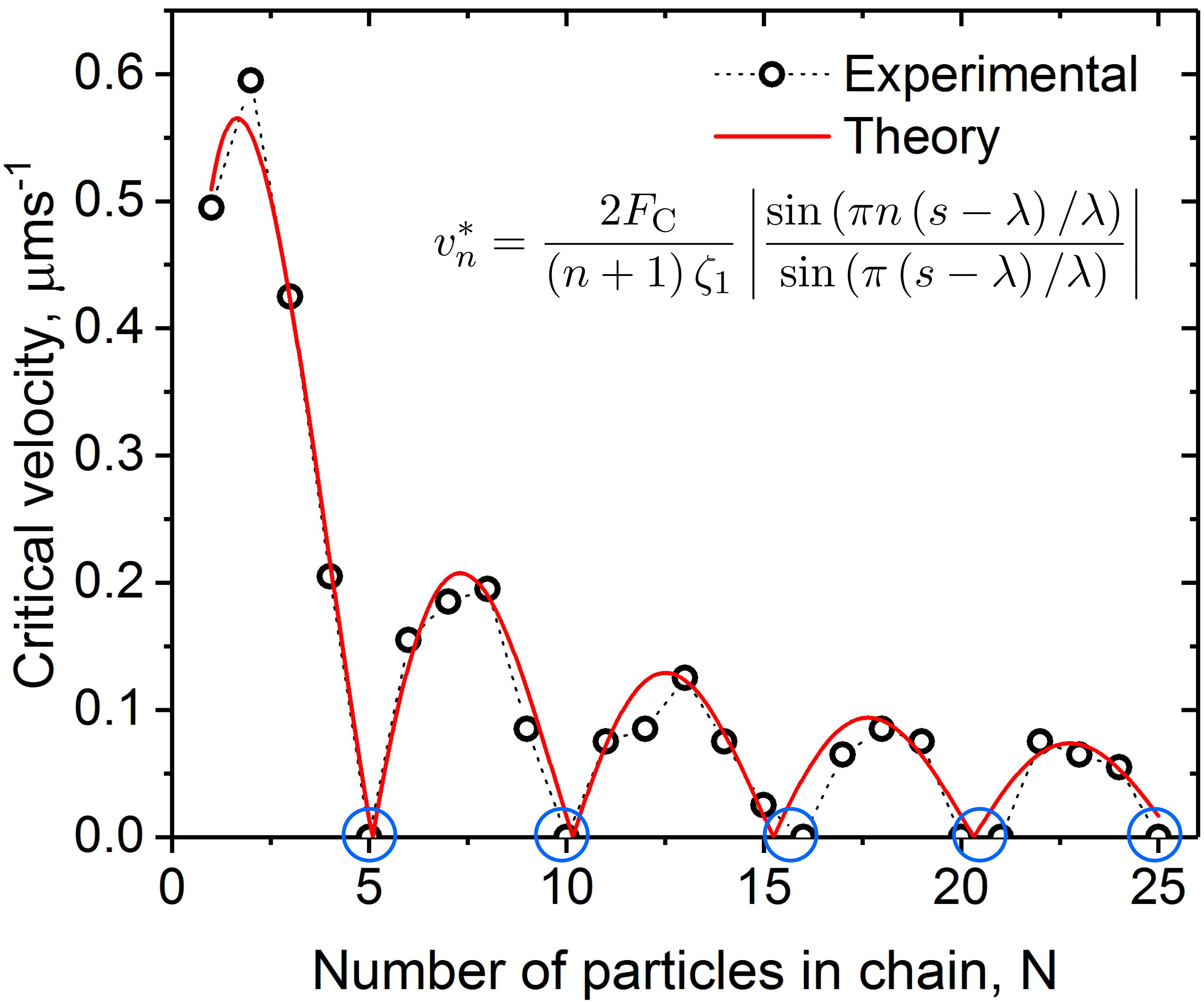
Potential 'depth' as a function of *N*





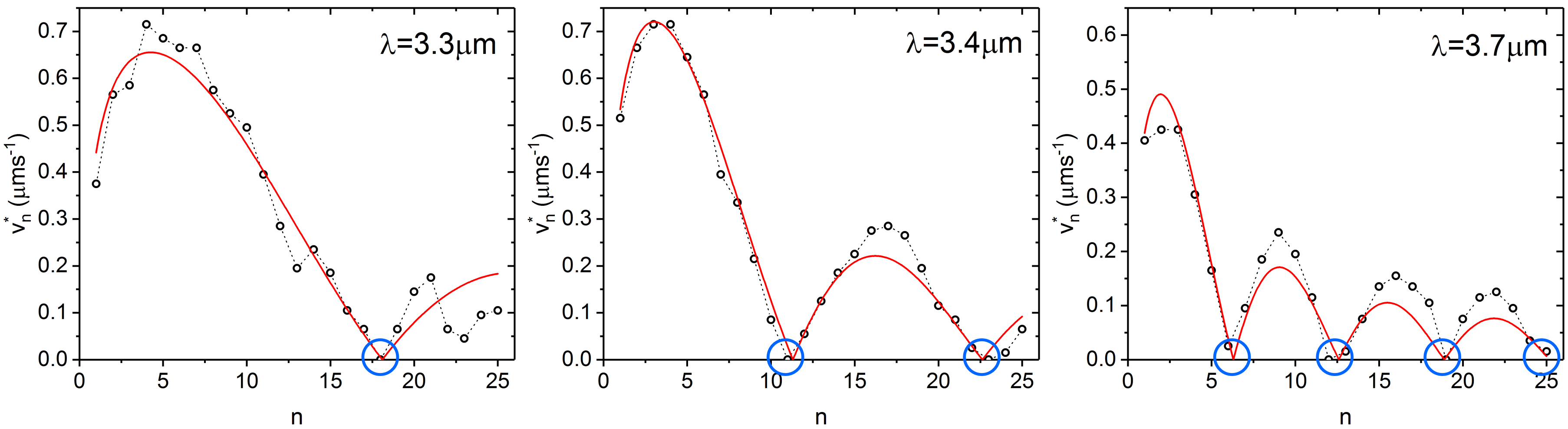
- *Periodic vanishing of critical velocity*

Critical velocity for *N* up to 25

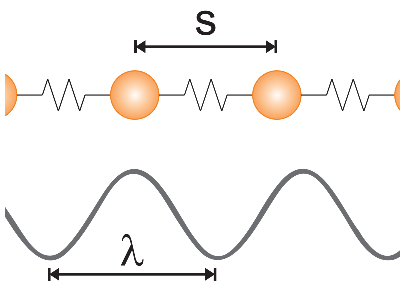


Different wavelengths of the landscape

$$v_n^* = \frac{2F_C}{(n+1)\,\zeta_1}\left|\frac{\sin\left(\pi n\left(s-\lambda\right)/\lambda\right)}{\sin\left(\pi\left(s-\lambda\right)/\lambda\right)}\right|$$



- Periodically **vanishing friction**
- Entirely determined by interplay of s and λ



Summary

- Driven colloid particles and chains in periodic optical potential energy landscapes show dynamic mode locking
- Colloidal model system allows access to microscopic details
- Phase portrait fingerprint of nature of the mode
- Periodically vanishing friction of finite colloidal chains

- *M.P.N. Juniper et al., Nature Commun. 6, 7187 (2015)*
- *M.P.N. Juniper et al., Phys. Rev. E 93, 012608 (2016)*
- *M.P.N. Juniper et al., New J. Phys. 19, 013010 (2017)*
- *J.L. Abbott et al., in preparation (2019)*

Thanks to:

- **Michael Juniper**, *University of Oxford*
- **Joshua Abbott**, *University of Oxford*
- **Urs Zimmermann**, *Heinrich-Heine University Duesseldorf*
- **Arthur Straube**, *Free University, Berlin, Germany*
- **Rut Besseling**, *InProcess-LSP, Oss, Netherlands*
- **Dirk Aarts**, *University of Oxford*
- **Hartmut Loewen**, *Heinrich-Heine University Duesseldorf*

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